|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Basic Integration Formulas:

## U Substitution

(Integration by substitution)

|  |  |
| --- | --- |
| Step 1: | Choose a u |
| Step 2: | Find du |
| Step 3: | Rewrite the integral in terms of u |
| Step 4: | Take the integral in terms of u |
| Step 5: | Substitute u back into the antiderivative in terms of x |
| Step 6: | If evaluating definite integral, you can either change the limits into terms of u or hold for the xs |

*The Chain rule is to differentiation as u-substitution is to integration*

**EXAMPLE 9:**

Find the average value of over the interval

First, set up the definition for average value and plug in and the interval

Find u and du

Rewrite in terms of u

Substitute u back into the antiderivative and solve as a normal definite integral

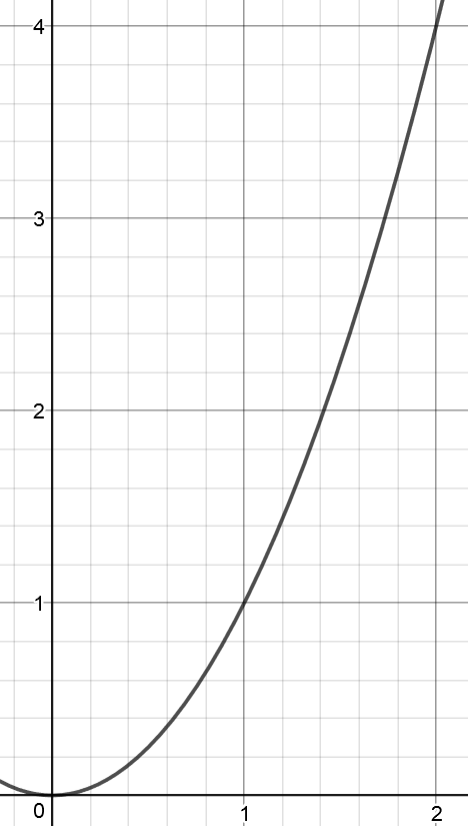
## Numerical Integration

Numerical integration involves approximating a definite integral through the use of shapes and functions with known areas.

**LRAM:**

Left Riemann Approximation Method: This method uses the left endpoint (the beginning) of each subinterval as the height for its rectangle.

What this looks like:



Here, we used 4 rectangles of width 0.5 to approximate . In this case, LRAM led to an underapproximation but that is not always the case and differs function to function.

1.125

0.5

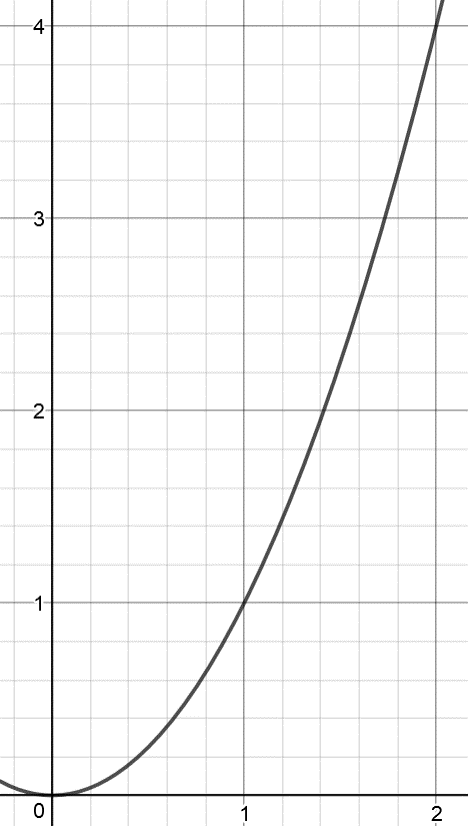
0

0.125

**RRAM:**

Right Riemann Approximation Method: This method uses the right endpoint (the end) of each subinterval as the height for its rectangle.

What this looks like:



Here, we used 4 rectangles of width 0.5 to approximate . In this case, RRAM led to an over-approximation but that is not always the case and differs function to function.

Notice that LRAM and RRAM only differ by 1 rectangle

1

1.125

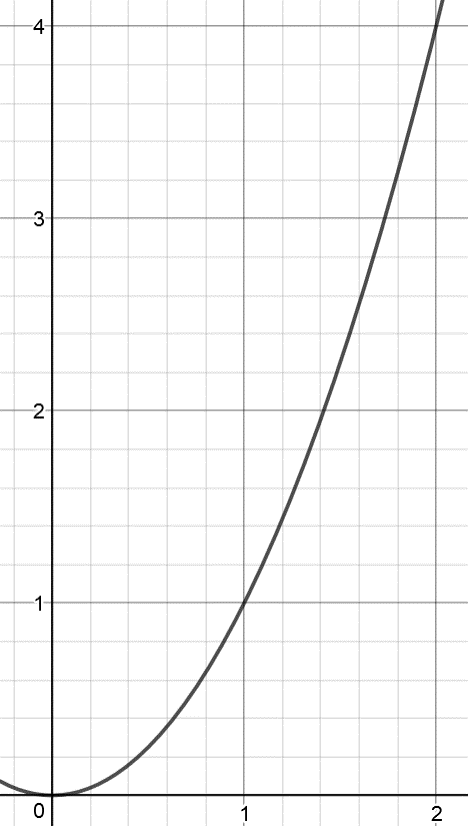
0.5

0.125

**MRAM:**

Mid Riemann Approximation Method or midpoint approximation: This method uses the midpoint (the middle) of each subinterval as the height for its rectangle.

What this looks like:



Here, we used 4 rectangles of width 0.5 to approximate . MRAM usually leads to a closer approximation than either

## The Trapezoid Rule

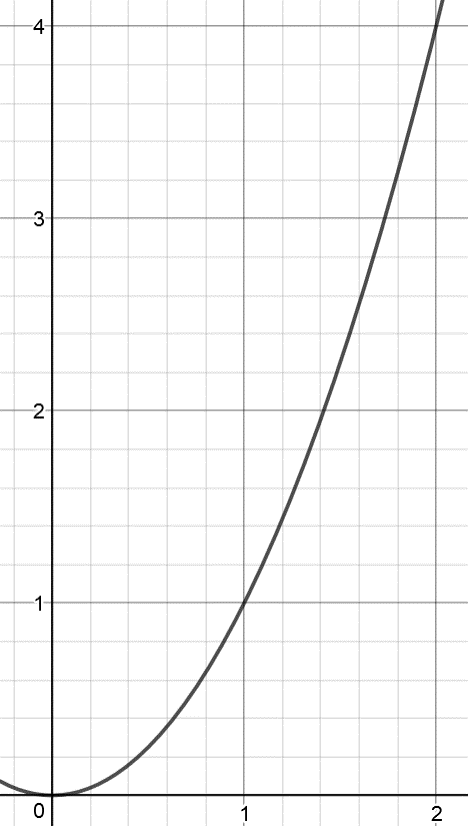
*Let be continuous on The trapezoidal rule for approximating is . As the approximation becomes more accurate.*

0.781

0.031

0.281

1.531



Here, we used 4 trapezoids of width 0.5 to approximate . The trapezoid rule is visually similar to LRAM with a right -hand triangle on top and is more accurate than LRAM or RRAM or MRAM

Note that the trapezoidal rule applies to intervals of equal width. You can still approximate using trapezoids but not using the formula previously defined. An example of this is shown below.

0.063

0.313

0.813

1.563